Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	7	6	/	0	1	Signature	

Paper Reference(s)

6676/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Friday 20 June 2008 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The variable y satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x + \cos y.$$

It is given that $y = 0.6$ at $x = 0$.

Use the approximation $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$, with a step length of 0.05, to estimate the values of y at $x = 0.05$ and $x = 0.1$, giving your answers to four decimal places.

(6)

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Question 1 continued

Q1

(Total 6 marks)

Turn over

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2.

$$\mathbf{M} = \begin{pmatrix} 1 & p & 2 \\ 0 & 3 & q \\ 2 & p & 1 \end{pmatrix},$$

where p and q are constants.

Given that $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is an eigenvector of **M**,

(a) show that $q = 4p$.

(3)

Given also that $\lambda = 5$ is an eigenvalue of **M**, and $p < 0$ and $q < 0$, find

(b) the values of p and q,

(4)

(c) an eigenvector corresponding to the eigenvalue $\lambda = 5$.

(3)

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Question 2 continued

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Question 2 continued

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Question 2 continued

(Total 10 marks)

Q2

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3. $$(x^2+1)\frac{d^2y}{dx^2} = 2y^2 + (1-2x)\frac{dy}{dx}. \qquad \text{(I)}$$

(a) By differentiating equation (I) with respect to x, show that

$$(x^2+1)\frac{d^3y}{dx^3} = (1-4x)\frac{d^2y}{dx^2} + (4y-2)\frac{dy}{dx}.$$

(3)

Given that $y = 1$ and $\frac{dy}{dx} = 1$ at $x = 0$,

(b) find the series solution for y, in ascending powers of x, up to and including the term in x^3.

(4)

(c) Use your series to estimate the value of y at $x = -0.5$, giving your answer to two decimal places.

(1)

Leave blank

Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

(Total 8 marks)

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4. The point P represents a complex number z on an Argand diagram such that

$$|z - 3| = 2|z|.$$

(a) Show that, as z varies, the locus of P is a circle, and give the coordinates of the centre and the radius of the circle.

(5)

The point Q represents a complex number z on an Argand diagram such that

$$|z + 3| = |z - \mathrm{i}\sqrt{3}|.$$

(b) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.

(5)

(c) On your diagram shade the region which satisfies

$$|z - 3| \geqslant 2|z| \quad \text{and} \quad |z + 3| \geqslant |z - \mathrm{i}\sqrt{3}|.$$

(2)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

(Total 12 marks)

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5. $$\mathbf{A} = \begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}, \text{ where } k \text{ is constant.}$$

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is represented by the matrix **A**.

(a) Find the value of k for which the line $y = 2x$ is mapped onto itself under T.
(3)

(b) Show that **A** is non-singular for all values of k.
(3)

(c) Find $\mathbf{A}^{-1}$ in terms of k.
(2)

A point P is mapped onto a point Q under T.

The point Q has position vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ relative to an origin O.

Given that $k = 3$,

(d) find the position vector of P.
(3)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 11 marks)

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6. De Moivre's theorem states that

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta \text{ for } n \in \mathbb{R}.$$

(a) Use induction to prove de Moivre's theorem for $n \in \mathbb{Z}^+$.

(5)

(b) Show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta .$$

(5)

(c) Hence show that $2\cos\frac{\pi}{10}$ is a root of the equation

$$x^4 - 5x^2 + 5 = 0 .$$

(3)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 13 marks)

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7.

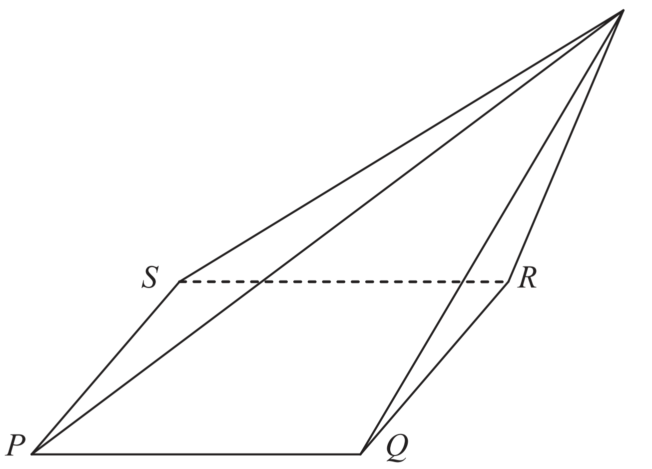

Figure 1

Figure 1 shows a pyramid *PQRST* with base *PQRS*.

The coordinates of *P*, *Q* and *R* are *P* (1, 0, –1), *Q* (2, –1, 1) and *R* (3, –3, 2).

Find

(a) $\overrightarrow{PQ} \times \overrightarrow{PR}$,
(3)

(b) a vector equation for the plane containing the face *PQRS*, giving your answer in the form **r . n** = *d*.
(2)

The plane *Π* contains the face *PST*. The vector equation of *Π* is **r . (i** – 2**j** – 5**k)** = 6.

(c) Find cartesian equations of the line through *P* and *S*.
(5)

(d) Hence show that *PS* is parallel to *QR*.
(2)

Given that *PQRS* is a parallelogram and that *T* has coordinates (5, 2, –1),

(e) find the volume of the pyramid *PQRST*.
(3)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

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